LESSON 8.6

Key Skills

Find the inverse of a quadratic function.

Find the inverse of $y = x^2 - 7x + 10$. Interchange x and y, and solve for y by applying the quadratic formula.

$$x = y^2 - 7y + 10$$

$$y = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(10 - x)}}{2(1)}$$

$$y = \frac{7 \pm \sqrt{9 + 4x}}{2}$$

Describe the transformations applied to the square-root parent function, $f(x) = \sqrt{x}$.

Describe the transformations applied to $f(x) = \sqrt{x}$ to obtain $y = 2\sqrt{3x - 3} + 4$.

$$g(x) = 2\sqrt{3x - 3} + 4 = 2\sqrt{3(x - 1)} + 4$$

The parent function is stretched vertically by a factor of 2, compressed horizontally by a factor of $\frac{1}{3}$, translated horizontally 1 unit to the right, and translated vertically 4 units up.

Exercises

Find the inverse of each quadratic function.

45. $y = 3x + x^2$ **46.** $y = 8x + 12 + x^2$

47. $y = 3x^2 - 16x + 5$ **48.** $y = 2x^2 + 7x + 6$

For each function, describe the transformations applied to $f(x) = \sqrt{x}$.

49. $g(x) = \frac{1}{3}\sqrt{x}$ **50.** $h(x) = 3\sqrt{x} - 5$

51. $k(x) = \sqrt{2x - 3}$ **52.** $g(x) = 4\sqrt{2x + 1} + 2$

53. $h(x) = -2\sqrt{3x} - 6$ **54.** $r(x) = 5\sqrt{3(x - 1)} + 1$

Evaluate each expression.

55. $5\left(\sqrt[3]{-27}\right)^2$ **56.** $\frac{1}{2}\sqrt[3]{8} + 1$

LESSON 8.7

Key Skills

Simplify expressions involving radicals.

Simplify $\dfrac{(24a^8b^5)^{\frac{1}{4}} \cdot \sqrt[4]{4a^3b^2}}{\sqrt[4]{3ab^2}}$.

$$\frac{(24a^8b^5)^{\frac{1}{4}} \cdot \sqrt[4]{4a^3b^2}}{\sqrt[4]{3ab^2}} = \frac{\sqrt[4]{24a^8b^5} \cdot \sqrt[4]{4a^3b^2}}{\sqrt[4]{3ab^2}}$$

$$= \frac{\sqrt[4]{96a^{11}b^7}}{\sqrt[4]{3ab^2}}$$

$$= \sqrt[4]{32a^{10}b^5}$$

$$= \sqrt[4]{2^4a^8b^4 \cdot 2a^2b}$$

$$= 2a^2|b|\sqrt[4]{2a^2b}$$

Rationalize the denominators of expressions.

Write $\dfrac{1}{2 + \sqrt{2}}$ with a rational denominator.

$$\frac{1}{2 + \sqrt{2}} = \frac{1}{2 + \sqrt{2}}\left(\frac{2 - \sqrt{2}}{2 - \sqrt{2}}\right) = \frac{2 - \sqrt{2}}{2}$$

Exercises

Simplify each radical expression. Assume that the value of each variable is positive.

57. $\sqrt{6x^2y^4} \cdot (3x^5y)^{\frac{1}{2}}$ **58.** $(5a^3b^5)^{\frac{1}{3}} \cdot \sqrt[3]{4a^4b}$

59. $\dfrac{\sqrt[3]{42c^4d^{17}}}{(6cd^{11})^{\frac{1}{3}}}$ **60.** $\dfrac{(45s^3t^6)^{\frac{1}{2}}}{\sqrt{3t^2}}$

61. $\dfrac{(6x^5y^7)^{\frac{1}{2}} \cdot \sqrt{3x^2y^4}}{\sqrt{2x}}$ **62.** $\dfrac{(24m^9n)^{\frac{1}{3}} \cdot \sqrt[3]{9m^3n^7}}{\sqrt[3]{3mn^2}}$

Write each expression with a rational denominator and in simplest form.

63. $\dfrac{1}{\sqrt{5}}$ **64.** $\dfrac{1}{\sqrt{7}}$

65. $\dfrac{3}{2 - \sqrt{3}}$ **66.** $\dfrac{4}{-2 + \sqrt{5}}$

67. $\dfrac{1 + \sqrt{2}}{3 - \sqrt{3}}$ **68.** $\dfrac{2 - \sqrt{3}}{3 + \sqrt{2}}$

LESSON 8.8

Key Skills

Solve radical equations.

Solve $2x = \sqrt{3 - x}$.

$$2x = \sqrt{3 - x}$$
$$4x^2 = 3 - x$$
$$4x^2 + x - 3 = 0$$
$$(4x - 3)(x + 1) = 0$$
$$x = \frac{3}{4} \quad or \quad x = -1$$

Check for extraneous solutions.

$2x = \sqrt{3 - x}$ $\qquad$ $2x = \sqrt{3 - x}$

$2\left(\frac{3}{4}\right) \stackrel{?}{=} \sqrt{3 - \left(\frac{3}{4}\right)}$ $\qquad$ $2(-1) \stackrel{?}{=} \sqrt{3 - (-1)}$

$\frac{3}{2} = \frac{3}{2}$ **True** $\qquad$ $-2 = 2$ **False**

Solve radical inequalities.

To solve $\sqrt{2x - 1} \leq 1$, first solve $2x - 1 \geq 0$.

$$2x - 1 \geq 0$$
$$x \geq \frac{1}{2}$$

Then solve the original inequality.

$$\sqrt{2x - 1} \leq 1$$
$$\left(\sqrt{2x - 1}\right)^2 \leq 1^2$$
$$2x - 1 \leq 1$$
$$2x \leq 2$$
$$x \leq 1$$

Thus, $x \geq \frac{1}{2}$ *and* $x \leq 1$, or $\frac{1}{2} \leq x \leq 1$. The solution can be verified by graphing.

Exercises

Solve each radical equation by using algebra. If the inequality has no real solution, write *no solution*. Check your solution.

69. $\sqrt{x + 2} = -2$ $\qquad$ **70.** $3\sqrt{x + 7} + 8 = 6$

71. $\sqrt[3]{x + 2} = -2$ $\qquad$ **72.** $3\sqrt[3]{x + 7} + 8 = 6$

73. ~~$\sqrt{x} = \sqrt{-2x}$~~ $\qquad$ **74.** $\sqrt{x + 2} = 3$

75. $\sqrt{x} = \sqrt{-x + 3}$ $\qquad$ **76.** $\sqrt{2x + 1} = \sqrt{4x - 4}$

77. $\sqrt[3]{4 - x} = \sqrt[3]{3x}$ $\qquad$ **78.** $\sqrt[5]{2x} = \sqrt[5]{x + 3}$

79. $\sqrt{x} - 2 = \sqrt{x - 2}$ $\qquad$ **80.** $\sqrt{3x} - 1 = \sqrt{x + 2}$

Solve each radical inequality by using algebra. Check your solution.

81. $\sqrt{x} \leq 5$ $\qquad$ **82.** $\sqrt{x - 1} < 2$

83. $\sqrt{x} \geq 5$ $\qquad$ **84.** $\sqrt{x - 1} > 2$

85. $\sqrt[4]{x - 2} \geq 1$ $\qquad$ **86.** $\sqrt[3]{x - 1} < 1$

87. $\sqrt{2x} + 2 > 4$ $\qquad$ **88.** $-2\sqrt{x - 2} < -1$

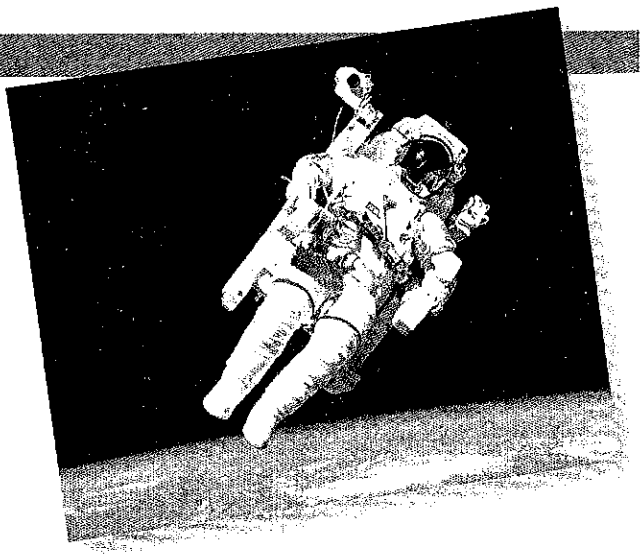
89. $\sqrt{6x} < 0$ $\qquad$ **90.** $4\sqrt{5x - 1} < 0$

Solve each radical inequality by graphing.

91 $\sqrt[3]{x - 2} \leq \sqrt{x}$ $\qquad$ **92** $\sqrt[5]{2x + 1} \geq 2$

Applications

PHYSICS The weight of an object varies inversely as the square of the distance from the object to the center of Earth, whose radius is approximately 4000 miles.



93. If an astronaut weighs 175 pounds on Earth, what will the astronaut weigh at a point 60 miles above Earth's surface?

94. If an astronaut weighs 145 pounds at a point 80 miles above the Earth's surface, how much does the astronaut weigh on Earth?

LESSON 8.7

Simplify each radical expression by using the Properties of *n*th Roots.

1. $\sqrt{125}$
2. $\sqrt[3]{162x^6y^3}$
3. $\sqrt[4]{80x^8z^{10}}$
4. $\sqrt[3]{-56x^4y^4z^3}$
5. $(75x^2y^3z)^{\frac{1}{2}}$
6. $(54x^5)^{\frac{1}{3}}$

Simplify each product or quotient. Assume that the value of each variable is positive.

7. $\sqrt[3]{9x^2} \cdot \sqrt[3]{3x}$
8. $\sqrt[3]{4x^5} \cdot \sqrt[3]{54xy^2}$
9. $\sqrt{8x^3} \cdot (2xz^5)^{\frac{1}{2}} \cdot \sqrt{4x^3z^4}$
10. $\dfrac{(81y^5)^{\frac{1}{4}}}{\sqrt[4]{x^4y}}$
11. $\dfrac{\sqrt[3]{48x^2y^4z^4}}{\sqrt[3]{6x}}$
12. $\dfrac{\sqrt{15x^9y^3}}{\sqrt{5x^5y}}$
13. $\sqrt[4]{8x^5} \cdot \sqrt[4]{4x^7}$
14. $\dfrac{\sqrt{9b^7}}{(12b^5)^{\frac{1}{2}}}$
15. $\dfrac{\sqrt[4]{8x^5}}{(20x^2)^{-\frac{1}{4}}}$

Find each sum, difference, or product. Give your answer in simplest radical form.

16. $(12 - \sqrt{2}) + (15 + \sqrt{2})$
17. $(9 + 2\sqrt{5}) - (1 + \sqrt{45})$
18. $(7 - 2\sqrt{6})(7 + 2\sqrt{6})$
19. $(3 - \sqrt{8})(5 + \sqrt{2})$
20. $(4 + \sqrt{3})(-2 + \sqrt{2})$
21. $6\sqrt{3}(2\sqrt{5} + 4\sqrt{6})$
22. $7\sqrt{20} + 8\sqrt{5} - 2\sqrt{45}$
23. $6\sqrt{8} - (\sqrt{24} - 3\sqrt{72} + \sqrt{54})$
24. $4\sqrt{2}(\sqrt{12} - 3\sqrt{2} + 4\sqrt{8})$
25. $(4\sqrt{2} - 2\sqrt{3})(5\sqrt{2} - \sqrt{3})$

Write each expression with a rational denominator and in simplest form.

26. $\dfrac{3}{\sqrt{15}}$
27. $\dfrac{\sqrt{135}}{\sqrt{15}}$
28. $\dfrac{5}{1 - \sqrt{6}}$
29. $\dfrac{-3}{\sqrt{6} - \sqrt{2}}$
30. $\dfrac{14}{\sqrt{5} + \sqrt{3}}$
31. $\dfrac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$
32. $\dfrac{2\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$
33. $\dfrac{2\sqrt{x}}{3\sqrt{x} - 4\sqrt{y}}$

LESSON 8.8

Solve each radical equation by using algebra. If the equation has no real solution, write *no solution*. Check your solutions.

1. $\sqrt{x - 5} = 3$
2. $\sqrt{x^2 - 15} = 7$
3. $\sqrt{x - 4} = \sqrt{x + 4}$
4. $\sqrt{2x - 5} + 4 = 3$
5. $\sqrt{3x - 5} = 5$
6. $\sqrt{5x - 11} = x - 1$
7. $\sqrt{2x - 1} = x$
8. $\sqrt[3]{x + 5} = \sqrt[3]{3x - 2}$
9. $\sqrt{x^2 - 4x - 5} = \sqrt{5x - x^2}$

Solve each radical inequality by using algebra. If the inequality has no real solution, write *no solution*. Check your solution.

10. $\sqrt{x - 3} \geq 2$
11. $3 > \sqrt{2x}$
12. $\sqrt{4x - 1} > 2$
13. $3 \geq \sqrt{x^2 - 4x + 4}$
14. $\sqrt{1 - x} > 3$
15. $\sqrt{3x - 2} \leq 2$
16. $4 \leq \sqrt{7 - x}$
17. $\sqrt{5x - 6} > 12$
18. $\sqrt{4x + 1} \geq 5$

Solve each radical equation or inequality by graphing. Round solutions to the nearest tenth. Check your solutions by any method.

19. $2\sqrt{x} \leq 3x - 4$
20. $3\sqrt{x + 2} \geq \sqrt{x^2 + 4}$
21. $0.25\sqrt{3x - 1} < x + 2$
22. $\sqrt[3]{x^2 + 1} = x$
23. $\sqrt[3]{x + 2} = \sqrt{x}$
24. $\sqrt[3]{2x - 1} > 2\sqrt{x - 4}$